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An Analytical Investigation of Finlines with Magnetized Ferrite Substrate

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Abstract —**This paper presents an analysis of a unilateral finline printed on a magnetized ferrite substrate. The network analysis method is applied to derive the determinantal equation. Numerical results are presented.**

I. INTRODUCTION

Recently, finlines have become attractive for millimeter-wave integrated-circuit application. Several papers have been published describing experimental and theoretical investigations for the various versions of finline structures printed on dielectric substrates [1]–[7]. Realization of nonreciprocal devices in finline techniques is also of interest in the millimeter-wave range because of the relative compactness and integrability of the devices compared to the nonreciprocal circuits built with conventional ferrite loaded waveguides. Beyer *et al.* [8], [9] have reported the experimental investigations of finline isolators and circulators. The theoretical treatment in [9] is also useful, however, which is based on TE-mode approximation. On the other hand, hybrid-mode analysis methods for the slot and striplines with magnetized ferrite substrates have been reported recently [10]–[12]. Lange [11] and Bock [12] employed the mode-matching procedure, while Mazur *et al.* [10] used the spectral-domain technique, a method which is superior to the former in numerical processing.

This paper presents an analysis method of the finline on a magnetized ferrite substrate. The method is based on the application of the network analysis techniques of electromagnetic fields [13], [14], which is similar to the spectral-domain technique from the viewpoint of applying Fourier transformation. Comparing this theory with the conventional spectral-domain techniques used in [10], however, the equivalent transmission-line concept is ideally suited for the analyses of the planar waveguide structures as indicated in [6], and the modal representation of the fields by TM-to-z, and TE-to-z modes (the z-axis is chosen perpendicular to the boundary surfaces of the substrates) facilitates the derivation of the equations.

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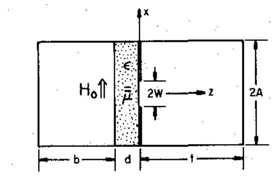

Fig. 1. Cross-sectional view of unilateral finline on ferrite substrate.

The determinantal equation for the propagation constant of a unilateral finline is obtained via matrix formulation in conjunction with Galerkin's procedure. Convergence checks are performed by increasing the number of basis functions for the representation of the aperture field. Some representative numerical results are included in the paper for the frequency range where the effective permeability $\mu_\perp > 0$, while the results for $\mu_\perp < 0$ are shown in [10]. The method is quite general and is applicable to other types of finline structures containing anisotropic media.

II. DETERMINANTAL EQUATION

The unilateral finline to be analyzed here is shown in Fig. 1, where the y-axis is chosen to be the direction of wave propagation. Since the dominant propagating mode of the finline is similar to the TE_{10} mode of the conventional rectangular waveguide, and the H-field near the slot is elliptically polarized, the ferrite slab should be magnetized in a direction parallel to the x-axis to realize efficient nonreciprocal circuits. When a ferrite sample is magnetized to saturation along the x-axis, the dyadic permeability of the ferrite is given by

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu & -jK \\ 0 & jK & \mu \end{bmatrix} \tag{1}$$

where μ_0 is the permeability of free space, and μ and K are dependent on the operating frequency ω, the applied dc magnetic field H_0, and the magnetization of the ferrite $4\pi M_s$.

As a first step toward deriving the determinantal equation, we express $\bar{E}_t$ and $\bar{H}_t$, the fields transverse to the z-axis, via the following Fourier integral:

$$\begin{Bmatrix} \bar{E}_t \\ \bar{H}_t \end{Bmatrix} = \frac{1}{\sqrt{2\pi}} \sum_{l=1}^{2} \sum_{m=0}^{\infty} \cdot \int_{-\infty}^{\infty} \begin{Bmatrix} V_m^{(l)}\ (\beta:z)\ \bar{f}_m^{(l)}\ (\beta:x) \\ I_m^{(l)}\ (\beta:z)\ \bar{g}_m^{(l)}\ (\beta:x) \end{Bmatrix} e^{-j\beta y} d\beta \tag{2}$$

where $l=1$ and $l=2$ represent the E-waves ($H_z \equiv 0$) and the H-waves ($E_z \equiv 0$), respectively. The vector mode functions $\bar{f}_m^{(l)}$ and $\bar{g}_m^{(l)}$ are given in Appendix I. They satisfy the boundary conditions at $x = \pm A$ and have the following orthonormal properties:

$$\int_{-A}^{A} \bar{f}_m^{(l)} \cdot \bar{f}_{m'}^{(l')*}\, dx = \int_{-A}^{A} \bar{g}_m^{(l)} \cdot \bar{g}_{m'}^{(l')*}\, dx = \delta_{ll'} \delta_{mm'} \tag{3}$$

where $\delta_{kk'}$ is Kronecker's delta, and the asterisk denotes a complex conjugate. Also, in (2) V_m and I_m are the modal voltages and currents. The longitudinal field components are derivable from the transverse components via the relationships

$$E_z = \frac{1}{j\omega\epsilon}\nabla\cdot(\bar{H}_t \times \hat{z}_0)$$
$$H_z = \frac{1}{j\omega\mu_{33}}\left[\nabla\cdot(\hat{z}_0 \times \bar{E}_t) - j\omega\hat{z}_0\cdot\bar{\bar{\mu}}\cdot\bar{H}_t\right] \quad (4)$$

where ϵ is the dielectric constant of the ferrite, μ_{33} is the $z-z$ component of $\bar{\bar{\mu}}$, which is identical to μ in the present problem, and $\hat{z}_0$ is the unit vector along the z-axis. Substituting (2) and (4) together with (1) into Maxwell's field equations, we obtain the transmission-line equations for the E- and H-mode amplitudes

$$-\frac{dV_m^{(l)}}{dz} = \sum_{l'=1}^{2}\left[ja_m^{(ll')}I_m^{(l')} + b_m^{(ll')}V_m^{(l')}\right]$$
$$-\frac{dI_m^{(l)}}{dz} = \sum_{l'=1}^{2}\left[jc_m^{(ll')}V_m^{(l')} + d_m^{(ll')}I_m^{(l')}\right] \quad (5)$$

where $a_m^{(ll')}$, $b_m^{(ll')}$, $c_m^{(ll')}$, and $d_m^{(ll')}$ are given in Appendix II. Replacement of ϵ by permittivity of free space ϵ_0, μ by μ_0, and K by 0 in Appendix II yields the transmission-line equations in the air region: $0 \leqslant z \leqslant t, -b-d \leqslant z \leqslant -d$.

The boundary conditions to be satisfied on the z-axis are expressed as

$$\bar{E}_t|_{z=t} = \bar{E}_t|_{z=-d-b} = 0 \qquad (-A \leqslant x \leqslant A) \quad (6a)$$
$$\bar{E}_t|_{z=-d-0} = \bar{E}_t|_{z=-d+0} \qquad (-A \leqslant x \leqslant A) \quad (6b)$$
$$\bar{H}_t|_{z=-d-0} = \bar{H}_t|_{z=-d+0} \qquad (-A \leqslant x \leqslant A) \quad (6c)$$
$$\bar{E}_t|_{z=-0} = \bar{E}_t|_{z=+0} \qquad (-A \leqslant x \leqslant A) \quad (6d)$$
$$\bar{H}_t|_{z=-0} = \bar{H}_t|_{z=+0} \qquad (-w \leqslant x \leqslant w). \quad (6e)$$

The above equations (6a)–(6d) can be replaced by the following continuity conditions of the modal voltages and currents:

$$V_m^{(l)}(t) = V_m^{(l)}(-b-d) = 0 \quad (7)$$
$$V_m^{(l)}(-d+0) = V_m^{(l)}(-d-0) \quad (8)$$
$$I_m^{(l)}(-d+0) = I_m^{(l)}(-d-0) \quad (9)$$
$$V_m^{(l)}(+0) = V_m^{(l)}(-0). \quad (10)$$

From (2) and (3), $V_m^{(l)}(0)$ is expressed in terms of the transverse electric field on the slot aperture $\bar{E}_0$ as

$$V_m^{(l)}(0) = \frac{1}{\sqrt{2\pi}}\int_{-w}^{w}dx'\int_{-\infty}^{\infty}dy'\,\bar{g}_m^{(l)}\cdot(\hat{z}_0 \times \bar{E}_0)e^{j\beta y'}. \quad (11)$$

Applying the continuity conditions given by (7)–(10) to the general solutions of the transmission-line equations and using the relation of (11), the modal voltages and currents are obtained in each of the subregions $0 \leqslant z \leqslant t$, $-d \leqslant z \leqslant 0$, and $-b-d \leqslant z \leqslant -d$. Substituting the results into (2) and (4), the electromagnetic fields can be expressed in terms $V_m^{(l)}(0)$, or implicitly, by the magnetic current distributions: $M = \hat{z}_0 \times \bar{E}_0$. Application of the remaining continuity condition (6e) yields the following integral equations:

$$\sum_{l=1}^{2}\sum_{l'=1}^{2}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty}d\beta\int_{-\infty}^{\infty}dy'\int_{-w}^{w}dx'\,p_m^{(ll')}(\beta)\bar{g}_m^{(l)}(\beta:x)\bar{g}_m^{(l')*}$$
$$\cdot(\beta:x')\cdot\bar{M}(x',y')e^{-j\beta(y-y')} = 0 \quad (12)$$

where $p_m^{(ll')} = Y_m^{(ll')}(+0) - Y_m^{(ll')}(-0)$, $Y_m^{(ll')}(+0)$ and $Y_m^{(ll')}(-0)$ are the current responses of the l-waves, respectively, at $z=+0$ and $z=-0$, due to the unit voltage source of the l'-wave at $z=0$ specifically

$$Y_m^{(ll')}(\pm 0) = I_m^{(l)}(\pm 0)/V_m^{(l')}(0). \quad (13)$$

Let β_0 be the desired propagation constant. The magnetic current may be expressed as

$$\bar{M}(x',y') = \bar{M}(x')e^{-j\beta_0 y'}. \quad (14)$$

Substituting (14) into (12) and integrating the results, we obtain

$$\sum_{l=1}^{2}\sum_{l'=1}^{2}\sum_{m=0}^{\infty}\int_{-w}^{w}p_m^{(ll')}(\beta_0)\bar{g}_m^{(l)}(\beta_0:x)\bar{g}_m^{(l')}(\beta_0:x')$$
$$\cdot\bar{M}(x')\,dx' = 0. \quad (15)$$

The determinantal equation can be obtained by an application of the Galerkin's method to the integral equation (15). We expand the unknown magnetic current in terms of a set of known basis functions ξ_n and η_n as follows:

$$\bar{M}(x') = \hat{x}_0\sum_{n'=1}^{2N_x}a_{n'}\xi_{n'}(x') + \hat{y}_0\sum_{n'=1}^{2N_y}jb_{n'}\eta_{n'}(x') \quad (16)$$

where $\hat{x}_0$ and $\hat{y}_0$ are the unit vectors along the x- and y-axes, respectively. Next we substitute (16) into (15) and take the inner product of the resulting equations with $\hat{x}_0\xi_n(x)$ and $\hat{y}_0\eta_n(x)$. This step yields a homogeneous matrix equation for the unknown expansion coefficients a_n and b_n as

$$\sum_{n'=1}^{2N_x}F_{nn'}^{(xx)}a_{n'} + \sum_{n'=1}^{2N_y}F_{nn'}^{(xy)}b_{n'} = 0 \quad (17a)$$
$$\sum_{n'=1}^{2N_x}F_{nn'}^{(yx)}a_{n'} + \sum_{n'=1}^{2N_y}F_{nn'}^{(yy)}b_{n'} = 0. \quad (17b)$$

Equations (17a) and (17b) are valid for $n = 1,2,\cdots,2N_x$ and $n = 1,2,\cdots,2N_y$, respectively. For (17) to yield a nontrivial solution, the determinant of the coefficient matrix associated with (17) must be zero. This condition results in the determinantal equation or the propagation constant

$$\mathrm{DET}\left\{\begin{bmatrix}F^{(xx)} & F^{(xy)}\\ F^{(yx)} & F^{(yy)}\end{bmatrix}\right\} \quad (18)$$

where $F^{(xx)}$, $F^{(xy)}$, $F^{(yx)}$, and $F^{(yy)}$ are $(2N_x \times 2N_x)$, $(2N_x \times 2N_y)$, $(2N_y \times 2N_x)$, and $(2N_y \times 2N_y)$ matrices, whose elements are $F_{nn'}^{(xx)}$, $F_{nn'}^{(xy)}$, $F_{nn'}^{(yx)}$, and $F_{nn'}^{(yy)}$, respectively (see Appendix III).

The final step is the choice of the basis functions. It is desirable that the edge effect of the aperture fields be accounted for, the aperture fields be systematically improved by increasing the number of basis functions, and that the integration of (A2) be performed analytically. These requirements prompt us to adopt the following families of functions:

$$\xi_n = U_n\left(\frac{x}{w}\right)$$
$$\eta_n = T_{n-1}\left(\frac{x}{w}\right)\Big/\sqrt{1-\left(\frac{w}{x}\right)^2} \quad (19)$$

where T_n and U_n are Chebyshev's polynomials of the first and second kind, respectively.

III. Numerical Computation

Since the finline analyzed here is loaded symmetrically with respect to the z-axis of the ferrite substrate geometry, the (xy), (yx), (xz), and (zx) components of $\bar{\bar{\mu}}$ are zero, and field can be separated into two modes, namely, the even and odd modes

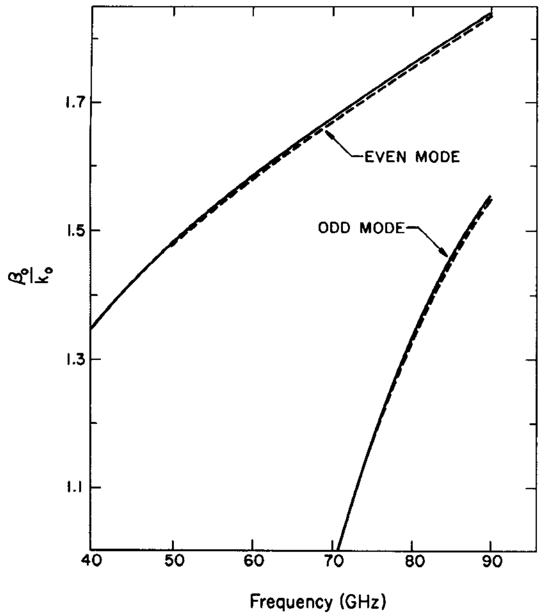

Fig. 2. Dispersion characteristics of even and odd modes propagating in the positive y-direction for a finline on ferrite and dielectric substrates ($\epsilon_r = 12.5$, $d = 0.005$ in, $2A = t = 0.094$ in, $b = 0.089$ in, $A/w = 2.0$. ----: $4\pi M_s = 5000$ [Ga], $H_0 = 500$ [O_e], ——: $K = 0$, $\mu = \mu_0$).

TABLE I
CONVERGENCE BEHAVIOR OF PHASE CONSTANT FOR FORWARD WAVES

N ($N_x = N_y$)	Normalized propagation const. β_0/k_0 even	odd
1	2.026	1.585
2	1.835	1.546
3	1.834	1.546

$\epsilon_r = 12.5$, $4\pi M_s = 5000$ [Ga], $H_0 = 500$ [0_e], $2A = t = 0.094$ in, $b = 0.089$ in, $d = 0.005$ in, $w = 0.0235$ in, freq. = 90 GHz.

which correspond to even and odd values of m in (2) or Appendix III. With this separation, (18) becomes

$$\text{DET}\left\{\begin{bmatrix} F_e^{(xx)} & F_e^{(xy)} \\ F_e^{(yx)} & F_e^{(yy)} \end{bmatrix}\right\} = 0 \qquad \text{DET}\left\{\begin{bmatrix} F_0^{(xx)} & F_0^{(xy)} \\ F_0^{(yx)} & F_0^{(yy)} \end{bmatrix}\right\} = 0. \tag{20}$$

Some results of the computation of the propagation constant for even and odd modes are given in Table I. Table I shows the comparison of the results for the dominant even and odd modes propagating in the positive y-direction obtained by using different expansion numbers. Note that the convergence for both the even and odd modes is quite rapid.

Fig. 2 shows the dispersion characteristics of the dominant even and odd modes for a finline on magnetized ferrite with a WR-19 waveguide shield. The characteristics of a dielectric loaded finline in which the magnetized ferrite is replaced by an isotropic substrate ($\mu = \mu_0$, $K = 0$) are also shown in Fig. 2. In view of the convergence characteristics of the numerical results exhibited in Table I, the number of the expansion functions (N_x, N_y) is chosen to be 2. Note that the difference between the forward and backward propagating waves for the thin ferrite substrate case is fairly small. Calculated examples for a thick ferrite substrate are shown in Fig. 3. It is observed that even for a thicker substrate the nonreciprocal effect is not substantial. In the above calculations, μ and K are assumed to be given by the following expressions:

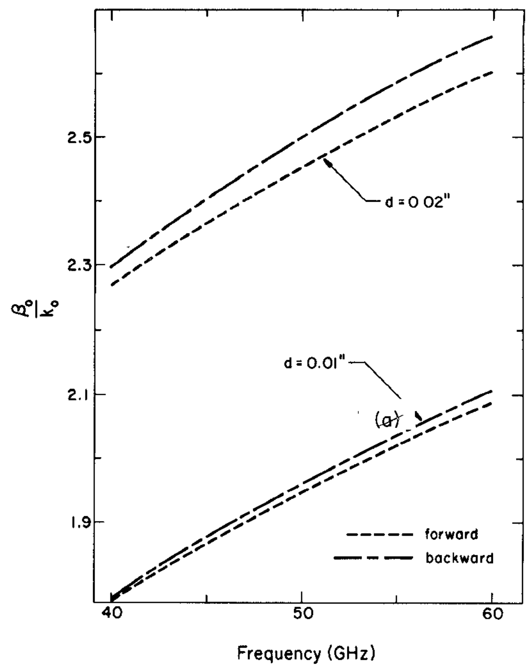

Fig. 3. Dispersion characteristics of even-mode propagation in the positive and negative y-directions ($\epsilon_r = 12.5$, $4\pi M_s = 5000$ [Ga], $H_0 = 500$ [0_e], $2A = t = 0.094$ in, $b = 0.094$ in $- d$, $A/w = 2.0$).

$$\frac{\mu}{\mu_0} = \frac{\omega^2 - \gamma H_0(\gamma H_0 + \gamma 4\pi M_s)}{\omega^2 - (\gamma H_0)^2}$$
$$\frac{K}{\mu_0} = \frac{|\gamma| 4\pi M_s \omega}{\omega^2 - (\gamma H_0)^2} \tag{21}$$

where γ is the gyro-magnetic ratio.

IV. CONCLUSIONS

The unilateral finline on a magnetized ferrite substrate has been analyzed using the network analysis technique for electromagnetic fields applied in conjunction with the Galerkin procedure. Some representative numerical solutions for the propagation constant are presented. It is found that the finline structure containing a single ferrite substrate does not exhibit adequate nonreciprocal characteristics. Beyer *et al.* [7] have reported that it is necessary to insert a spacer between the metal fin and the ferrite substrate in order to realize efficient nonreciprocal devices. The analysis of such multilayered finlines with ferrites is beyond the scope of this paper; however, the problem is currently being investigated by the authors both theoretically and experimentally, and the results will be reported in a future publication.

APPENDIX I
VECTOR MODE FUNCTIONS

$$f_m^{(1)} = \sqrt{\frac{\epsilon_m}{2A}}\,\frac{1}{K_m}\left[\hat{x}_0 \gamma_m \cos\gamma_m(x+A) - \hat{y}_0 j\beta \sin\gamma_m(x+A)\right]$$

$$f_m^{(2)} = \sqrt{\frac{\epsilon_m}{2A}}\,\frac{1}{K_m}\left[\hat{x}_0 j\beta \cos\gamma_m(x+A) - \hat{y}_0 \gamma_m \sin\gamma_m(x+A)\right]$$

$$g_m^{(l)} = \hat{z}_0 \times f_m^{(l)}$$

$$\epsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \geq 1 \end{cases}$$

$$K_m^2 = \gamma_m^2 + \beta^2 \qquad \gamma_m = \frac{m\pi}{2A}.$$

Appendix II
Coefficients of the Transmission-Line Equation in the Ferrite Loaded Region

$$a_m^{(11)} = \frac{\omega\left(\mu_0\beta^2 + \mu_\perp \gamma_m^2\right)}{K_m^2} - \frac{K_m^2}{\omega\epsilon}$$

$$a_m^{(12)} = -a_m^{(21)} = -j\beta\gamma_m\omega\left(\mu_0 - \mu_\perp\right)/K_m^2$$

$$a_m^{(22)} = \omega\left(\mu_0\gamma_m^2 + \mu_\perp\beta^2\right)/K_m^2$$

$$b_m^{(12)} = d_m^{(21)} = -j\frac{K}{\mu}\gamma_m$$

$$b_m^{(22)} = -d_m^{(22)} = -\frac{K}{\mu}\beta, \qquad b_m^{(ll)} = d_m^{(ll)} = 0$$

$$c_m^{(11)} = \omega\epsilon, \qquad c_m^{(12)} = c_m^{(21)} = 0$$

$$c_m^{(22)} = \omega\epsilon - \frac{K_m^2}{\omega\mu}$$

where

$$\mu_\perp = \mu - \frac{K^2}{\mu}.$$

Appendix III
Elements of the Determinant

$$F_{nn'}^{(xx)} = \sum_{m=1}^{\infty} \frac{1}{AK_m^2}\left[\beta_0^2\left(Y_{11}^+ - Y_{11}^-\right) - j\beta\gamma_m\left(Y_{12}^- - Y_{21}^-\right) + \gamma_m\left(Y_{22}^+ - Y_{22}^-\right)\right]\hat{\xi}_n\hat{\xi}_{n'}$$

$$F_{nn'}^{(xy)} = \sum_{m=1}^{\infty} \frac{1}{AK_m^2}\left[\beta\gamma_m\left(-Y_{11}^+ + Y_{11}^- + Y_{22}^+ - Y_{22}^-\right) - j\beta^2 Y_{12}^- + j\gamma_m^2 Y_{21}^-\right]\tilde{\xi}_n\tilde{\eta}_{n'}$$

$$F_{nn'}^{(yx)} = \sum_{m=1}^{\infty} \frac{1}{Ak_m^2}\left[\beta\gamma_m\left(-Y_{11}^+ + Y_{11}^+ + Y_{22}^+ - Y_{22}^-\right) + j\gamma_m Y_{12}^- + j\beta^2 Y_{21}^-\right]\tilde{\eta}_n\tilde{\xi}_{n'}$$

$$F_{nn'}^{(yy)} = \sum_{m=0}^{\infty} \frac{\epsilon_m}{2AK_m^2}\left[\gamma_m^2\left(Y_{11}^+ - Y_{11}^-\right) + j\beta\gamma_m\left(Y_{12}^- - Y_{21}^-\right) + \beta^2\left(Y_{22}^+\right)\right]\tilde{\eta}_n\tilde{\eta}_{n'} \quad \text{(A1)}$$

where

$$Y_{ll'}^{\pm} = Y_m^{(ll')}(\pm 0)$$

$$\tilde{\xi}_n = \int_{-w}^{w} \sin\gamma_m(x+a)\,\xi_n(x)\,dx$$

$$\tilde{\eta}_n = \int_{-w}^{w} \cos\gamma_m(x+a)\,\eta_n(x)\,dx. \quad \text{(A2)}$$

Note that

$$Y_{12}^+ = Y_{21}^+ = 0.$$

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Propagation in Coupled Unilateral and Bilateral Finlines

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Abstract—**The propagation characteristics of coupled unilateral and bilateral finlines in the even and odd modes are evaluated with the hybrid-mode formulation in the spectral domain. The frequency-dependent guide wavelength is obtained from the solution of the characteristic equation. The characteristic impedance based on the power–voltage definition is also evaluated. Numerical results are presented to study the influence of various structural parameters on the characteristics of finlines.**

I. Introduction

Among various forms of transmission media, finlines have demonstrated potential for their use in millimeter-wave integrated circuits [1], [2]. In a typical finline configuration, a planar circuit is placed in the E-plane of a rectangular waveguide. This combines the advantages of both planar circuits and waveguides. Its wide single-mode bandwidth, low dispersion, moderate attenuation and compatibility with semiconductor devices are attractive features at millimeter wavelengths.

The propagation in finlines has been a subject of considerable interest recently. An accurate description of the dispersion can be obtained with the hybrid-mode formulations [3]–[5]. This was first presented by Hofmann [6] who used the space-domain formulation. Subsequently, the spectral-domain formulation was utilized by Knorr and Shayda [7], Schmidt and Itoh [8], as well as by the authors [9]–[12], to determine the propagation characteristics of unilateral and bilateral finlines.

With the increased interest in finlines, there is a need to

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